Summarizing and Visualizing Graph Ensembles with Rank Statistics and Boxplots

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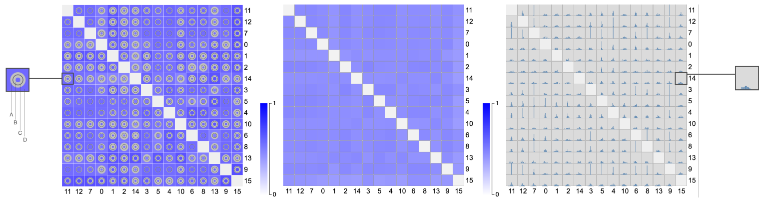
1 Introduction

The problem of visualizing graphs becomes more complex as we consider the growing diversity of visualization tasks on graphs. For instance, in the domain of neuroscience, there is a need to gain insight into how a graph (representing a brain network) compares to another, or how a graph ensemble (brain networks associated with a specific group) compares to an individual graph or another ensemble [1]. The goal of this project is to develop a method to visualize graph ensembles in a way that is able to convey the underlying structure (both the center and variability of the underlying distribution of edge weights) in context of the relationships between nodes in the graph. Specifically, we hope to help accomplish two important tasks that pertain to applications involving weighted graph ensembles: *1)* comparing two different graph ensembles and *2)* comparing individual members relative to an ensemble. We limit the scope of this project to graphs ensembles that share common vertex/edge sets and differ only with regard to edge weights.

2 Method

We propose a visualization method, called *network boxplot*, for visualizing graph ensembles. The first step for drawing a network boxplot, analogous to the traditional Tukey boxplot and other existing methods for summarizing ensembles [3–5], is to compute center outward order and rank statistics for the members in the ensemble. We use a discrete functional representation of graph adjacency matrices which allows us to use the functional band depth (see [2]) to obtain required statistics for members of the ensemble. The second step is to generate a visualization using the order and rank statistics. Fig. 1a shows a rendering of the network boxplot. We employ an adjacency matrix representation, and use cells in a *single* matrix to display the summary statistics for the ensemble. Each cell in a network boxplot encodes the weight on the median graph and the extent of weights on graphs in the 50 percent band for corresponding edges in the graph ensemble. The weight on median graph is encoded in two ways: the background color of the cell as well as the radius of circle between the two gray rings (annuli). The upper and lower extents of the 50 percent bands are encoded by the outer and inner gray rings.



(a) (b) (c)

Fig. 1. Visualizing graph ensembles. (a) A network boxplot visualization. ‘A’ and ‘B’ (anuli) indicate the 50 percent band while ‘C’ (circle radius) and ‘D’ (background color) are two different encodings of the median. (b) Heatmap, and (c) cell histogram for weighted adjacency matrix ensemble

3 Results and Discussion

We conducted a pilot user study to evaluate the performance of the proposed network boxplot visualization in the task of comparing two graph ensembles (edge weights were generated using Gaussian processes). We found that participants made more accurate—although slower— judgments using the proposed method relative to the existing methods—namely, heatmap (Fig. 1b) and cell histogram [6] (Fig. 1c). A key advantage of our approach over existing methods is the ability to convey correlations across edges. We plan to conduct a larger user study which would also include the task of comparing an individual graph to an ensemble. We have also developed a network boxplot based interactive system to explore real brain fMRI network ensemble data. Our plan is to work with domain experts to evaluate the system and improve its effectiveness.

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References

1. Alper, B., Bach, B., Henry Riche, N., Isenberg, T., Fekete, J.D.: Weighted graph comparison techniques for brain connectivity analysis. In: Proceedings of the SIGCHI Conference on Human Factors in Computing Systems. pp. 483–492. ACM (2013)
2. López-Pintado, S., Romo, J.: On the concept of depth for functional data. Journal of the American Statistical Association 104(486), 718–734 (2009)
3. Mirzargar, M., Whitaker, R., Kirby, R.: Curve boxplot: Generalization of boxplot for ensembles of curves. Visualization and Computer Graphics, IEEE Transactions on 20(12), 2654–2663 (Dec 2014)
4. Sun, Y., Genton, M.G.: Functional boxplots. Journal of Computational and Graphical Statistics 20(2), 316–334 (2011)
5. Whitaker, R.T., Mirzargar, M., Kirby, R.M.: Contour boxplots: A method for characterizing uncertainty in feature sets from simulation ensembles. IEEE Transactions on Visualization and Computer Graphics 19(12), 2713–2722 (Dec 2013)

6. Yi, J.S., Elmqvist, N., Lee, S.: Timematrix: Analyzing temporal social networks using interactive matrix-based visualizations. Intl. Journal of Human–Computer Interaction 26(11-12), 1031–1051 (2010)